PLANAR WAVES IN NONLINEAR VISCOUS MULTICOMPONENT MEDIA

V. V. Nefedov

UDC 624.131+532.529

One of the fundamental methods for the study of wave processes in multicomponent media is the method presented in [1] in which a multicomponent medium is regarded as a homogeneous continuous medium with an equation of compressibility taking compressibility into account and in which the components of the medium are in an equilibrium state. In [2] the model presented in [1] was improved by the introduction of nonlinear diagrams of dynamic and static compression of a multicomponent medium, thereby making it possible to introduce a volume viscosity; also introduced and studied in [2] were the fundamental results obtained in the solution of problems relating to the propagation of waves in media with constant and variable viscosity. In [3] the results of experimental studies of spherical waves in frozen soils with various physicomechanical characteristics were given; it was shown that the wave parameters depend essentially on the characteristics of the soil in its unfrozen state.

The study of wave processes in ice appears in [4-7, etc]. In [4] results of experimental studies are given based on the theory presented in [5]; these results are in the form of graphs showing the attenuation of the maximum value of the speed of ice particles as a function of the distance of the particles from the point where shock loading is applied.

On the basis of the experimental data in [6, 7] expressions were obtained describing the process of wave propagation in ice and an equation was given defining the behavior of ice as a viscoelastic medium.

In the present paper, based on results presented in [2], we obtain a solution of the problem of propagation of a planar wave produced by nonstationary loading in a multicomponent medium. In this connection, we took, a viscous component, ice, the compressibility equation for which we borrowed from [6]. The solution we obtained with the aid of a computer employed the method of characteristics, a method previously employed in [8] with nonviscous media and in [2] with media with volume viscosity.

We consider the problem of propagation of a plane wave in a viscous multicomponent medium. We employ a model containing three components: liquid, solid, and ice. As was done in [2], under initial (atmospheric) pressure p_0, we denote the volume content of ice, liquid, and solid components by α_i, specific volume by V_{i0}, density by ρ_{i0}, sound speed in each component by c_{i0}; finally, for the medium taken as a whole we denote its density by ρ_0 and its specific volume by V_0. We employ subscript i = 1 for ice, i = 2 for the liquid component, and i = 3 for the solid component. When the pressure is p we denote the volume, density, and speed of sound in the components by V_i, ρ_i, and c_i, respectively; we denote the density of the medium by ρ and its specific volume by V:

$$\rho_0 = \frac{1}{V_0} = \sum_{i=1}^{3} \alpha_i \rho_{i0}, \quad \sum_{i=1}^{3} \alpha_i = 1. \tag{1}$$

The liquid and solid components, under the influence of a load, are compressed in accordance with the Tate equation

$$p - p_0 = \frac{\rho_{i0} c_{i0}^2}{\gamma_i} \left[\left(\frac{V_i}{V_{i0}} \right)^{-\gamma_i} - 1 \right], \tag{2}$$

where i = 2, 3; γ_i are known constants.

Compression of ice is determined from an equation given in [6], which, in our notation, may be written as follows:

$$\frac{\dot{p}}{E_D} + \frac{\dot{V}_1}{V_{10}} = -\frac{1}{\eta} \left(E_S \frac{V_1 - V_{10}}{V_{10}} + p - p_0 \right). \tag{3}$$

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 128-133, November-December, 1987. Original article submitted September 1, 1986.

0021-8944/87/2806-0919$12.50

Here E_D and E_S are the dynamic and static elasticity moduli; η is the coefficient of viscosity of ice.

From Eq. (3) we obtain

$$\frac{V_1}{V_{10}} = \frac{\eta}{E_S}\left(-\frac{\dot{p}}{E_D} - \frac{\dot{V}_1}{V_{10}} - \frac{p-p_0}{\eta} + \frac{E_S}{\eta}\right). \tag{4}$$

Taking note of the fact that

$$\frac{\rho_0}{\rho} = \frac{V}{V_0} = \sum_{i=1}^{3} \alpha_i \frac{V_i}{V_{i0}}, \tag{5}$$

we have, from Eqs. (2) and (4),

$$\frac{V}{V_0} = \alpha_1 \frac{\eta}{E_S}\left(-\frac{\dot{p}}{E_D} - \frac{\dot{V}_1}{V_{10}} - \frac{p-p_0}{\eta} + \frac{E_S}{\eta}\right) + \sum_{i=2}^{3} \alpha_i \left(\frac{p-p_0}{\rho_{i0}c_{i0}^2}\gamma_i + 1\right)^{-\frac{1}{\gamma_i}}. \tag{6}$$

Differentiating Eqs. (5) with respect to the time and substituting into it the values $\dot{V}_2$ and $\dot{V}_3$, determined from Eq. (2), and the value of $\dot{V}_1$ from Eq. (6), we obtain

$$\frac{\dot{V}}{V_0} = -\sum_{i=2}^{3}\left[\alpha_i \left(\frac{p-p_0}{\rho_{i0}c_{i0}^2}\gamma_i + 1\right)^{-\frac{1+\gamma_i}{\gamma_i}} \frac{\dot{p}}{\rho_{i0}c_{i0}^2}\right] -$$
$$\alpha_1 \left\{\left[\frac{V}{V_0} - \sum_{i=2}^{3}\alpha_i \left(\frac{p-p_0}{\rho_{i0}c_{i0}^2}\gamma_i + 1\right)^{-\frac{1}{\gamma_i}}\right]\frac{E_S}{\eta\alpha_1} + \frac{\dot{p}}{E_D} + \frac{p-p_0}{\eta} - \frac{E_S}{\eta}\right\} \tag{7}$$

or, finally,

$$\frac{\dot{V}}{V_0} = -\varphi(p)\,\dot{p} + \frac{\psi_1\left(p, \frac{V}{V_0}\right)}{\eta},$$
$$\varphi(p) = \sum_{i=2}^{3}\left[\alpha_i \left(\frac{p-p_0}{\rho_{i0}c_{i0}^2}\gamma_i + 1\right)^{-\frac{1+\gamma_i}{\gamma_i}} \frac{1}{\rho_{i0}c_{i0}^2}\right] + \frac{\alpha_1}{E_D}, \tag{8}$$
$$\psi_1\left(p, \frac{V}{V_0}\right) = -\left[\frac{V}{V_0} - \sum_{i=2}^{3}\alpha_i \left(\frac{p-p_0}{\rho_{i0}c_{i0}^2}\gamma_i + 1\right)^{-\frac{1}{\gamma_i}}\right]E_S - \alpha_1(p - p_0 - E_S).$$

The basic equations of motion in the Lagrange variables r, t have the form

$$\frac{\partial \varepsilon}{\partial t} - \frac{\partial v}{\partial r} = 0, \quad \frac{\partial v}{\partial t} + \frac{1}{\rho_0}\frac{\partial p}{\partial r} = 0, \tag{9}$$

where ε is the deformation, v is the speed of particles of the medium, p is the pressure, and ρ_0 is the initial density of the medium.

The closing Eq. (8) is transformed into

$$\frac{\partial \varepsilon}{\partial t} + \varphi(p)\frac{\partial p}{\partial t} = \frac{\psi(p, \varepsilon)}{\eta}, \quad \psi(p, \varepsilon) \equiv \psi_1(p, \varepsilon + 1). \tag{10}$$

The system (9), (10) is hyperbolic. In the (r, t)-plane there are three families of real characteristics r = r(t). The characteristic relations are as follows:

$$dp \pm \sqrt{\frac{\rho_0}{\varphi(p)}}\,dv = \frac{\psi(p, \varepsilon)}{\eta\varphi(p)}\,dt \text{ along the curves } \dot{r} = \pm\frac{1}{\sqrt{\rho_0\varphi(p)}}; \tag{11}$$

$$dp + \frac{1}{\varphi(p)}\,d\varepsilon = \frac{\psi(p, \varepsilon)}{\eta\varphi(p)}\,dt \text{ along the curves } \dot{r} = 0. \tag{12}$$

The boundary conditions at the initial section r = 0 and at the front of the wave [the precursor r = R(t) or t = T(r)] are as follows:

$$p(0, t) = \begin{cases} p_0 + p_m e^{-\frac{t}{\theta}}, & t \geqslant 0 \quad (\theta = \text{const} > 0), \\ p_0, & t < 0; \end{cases} \tag{13}$$

$$p - p_0 = \dot{R}\rho_0 v,\ v = -\dot{R}\varepsilon \text{ for } r = R(t) \tag{14}$$

($\dot{R}$ is the speed of the precursor).

To obtain the third relationship at the precursor front it is necessary to integrate Eq. (10), discarding the right-hand side [2]. For r = R(t)

$$\varepsilon = -\frac{\alpha_1}{E_D}(p - p_0) + \sum_{i=2}^{3}\left[\alpha_i\left(\frac{p - p_0}{\rho_{i0}c_{i0}^2}\gamma_i + 1\right)^{-\frac{1}{\gamma_i}} - \alpha_i\right]. \tag{15}$$

The viscosity coefficient η for ice, in accord with [6, 7], is given by the expression

$$\eta = A(t - T(r))^{2/3}, \tag{16}$$

where A is a known constant and t = T(r) is the equation of the precursor in the (r, t)-plane.

Solution of the problem reduces to integration of system (9), (10) subject to the boundary conditions (13) at the initial section and the conditions (14), (15) at the front of the wave.

We introduce dimensionless variables $p^0 = \frac{p - p_0}{p_m}$, $v^0 = \frac{c_0\rho_0}{p_m}v$, $\varepsilon^0 = -\frac{E_D}{p_m}\varepsilon$ and dimensionless Lagrange variables

$$r^0 = \frac{(\nu\alpha)^{-3}}{c_0}r,\ t^0 = (\nu\alpha)^{-3}t, \text{ where } \nu = E_D/E_S;\ \alpha = A/E_D;\ c_0 = \sqrt{E_D/\rho_0}.$$

In these variables, Eqs. (9), (10) have the form

$$\partial\varepsilon^0/\partial t^0 + \partial v^0/\partial r^0 = 0; \tag{17}$$

$$\partial v^0/\partial t^0 + \partial p^0/\partial r^0 = 0; \tag{18}$$

$$\partial\varepsilon^0/\partial t^0 - \varphi^0(p^0)\partial p^0/\partial t^0 = \psi^0(p^0,\ \varepsilon^0)/\eta^0. \tag{19}$$

Here $\varphi^0(p^0) = \sum_{i=2}^{3}\left[\alpha_i\left(\frac{p_m\gamma_i}{\rho_{i0}c_{i0}^2}p^0 + 1\right)^{-\frac{1+\gamma_i}{\gamma_i}}\frac{E_D}{\rho_{i0}c_{i0}^2}\right] + \alpha_1$; $\psi^0(p^0,\ \varepsilon^0) = \frac{\nu E_S}{p_m}\left[-\frac{p_m}{E_D}\varepsilon^0 + 1 - \sum_{i=2}^{3}\alpha_i\left(\frac{p_m\gamma_i}{\rho_{i0}c_{i0}^2}p^0 + 1\right)^{-\frac{1}{\gamma_i}} + \alpha_1\left(\frac{p_m}{E_S}p^0 - 1\right)\right]$; $\eta^0 = \left(t^0 - \frac{T\left(c_0(\nu\alpha)^3 r^0\right)}{(\nu\alpha)^3}\right)^{2/3} = (t^0 - T^0(r^0))^{2/3}$.

We may write boundary conditions (13)-(15) in dimensionless variables as follows:

$$p^0(0,\ t^0) = \begin{cases} e^{-\frac{(\nu\alpha)^3}{\theta}t^0}, & t^0 \geqslant 0, \\ 0, & t^0 < 0; \end{cases} \tag{20}$$

$$p^0 = \frac{\dot{R}}{c_0}v^0,\quad v^0 = \frac{\dot{R}}{c_0}\varepsilon^0,$$
$$\varepsilon^0 = \alpha_1 p^0 + \frac{E_D}{p_m}\left\{\sum_{i=2}^{3}\left[\alpha_i - \alpha_i\left(\frac{p_m\gamma_i}{\rho_{i0}c_{i0}^2}p^0 + 1\right)^{-\frac{1}{\gamma_i}}\right]\right\} \tag{21}$$

for $r^0 = R^0(t^0)$ [$t^0 = T^0(r^0)$].

Characteristic relations (11), (12) may be rewritten in the form

$$dp^0 \pm \frac{1}{\sqrt{\varphi^0(p^0)}}dv^0 = -\frac{\psi^0(p^0,\ \varepsilon^0)}{\eta^0\varphi^0(p^0)}dt^0 \text{ along } \dot{r}^0 = \pm\frac{1}{\sqrt{\varphi^0(p^0)}},$$

$$dp^0 - \frac{1}{\varphi^0(p^0)}d\varepsilon^0 = -\frac{\psi^0(p^0,\ \varepsilon^0)}{\eta^0\varphi^0(p^0)}dt^0 \text{ along } \dot{r}^0 = 0.$$

The system of equations (17)-(19) with the boundary conditions (20), (21) was solved with the aid of a computer by the method of characteristics. In accord with [3, 7] we took ρ_{20} = 1000 kg/m³, ρ_{30} = 2660 kg/m³, c_{20} = 1500 m/sec, c_{30} = 5000 m/sec, $p_m = 2\cdot 10^7$ N/m², $E_D = 10^{10}$ N/m², $E_S = 25\cdot 10^8$ N/m², θ = 1 sec, ν = 4, $(\nu\alpha)^3 = 10^5$ sec, $\gamma_2 = 7$, $\gamma_3 = 4$; α_i were the following: $\alpha_1 = 0.69$, $\alpha_2 = 0.01$, $\alpha_3 = 0.3$ and $\alpha_1 = 0.6$, $\alpha_2 = 0.1$, $\alpha_3 = 0.3$

The calculations show that a degeneracy of the viscosity coefficient η [see Eq. (16)] at the front of the wave leads to the result that a continuous wave of compression begins to propagate from the initial section at the instant of application of shock loading. Unlike

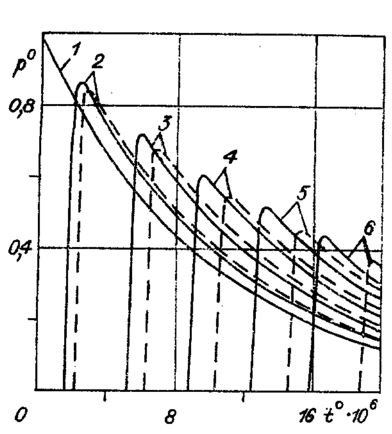

Fig. 1

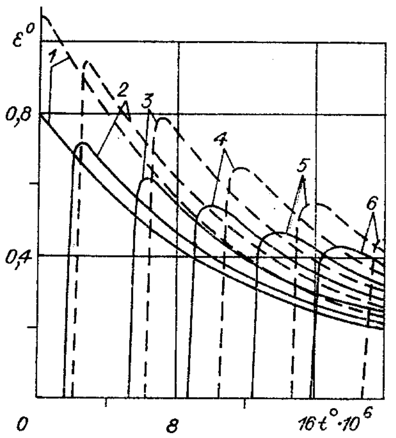

Fig. 2

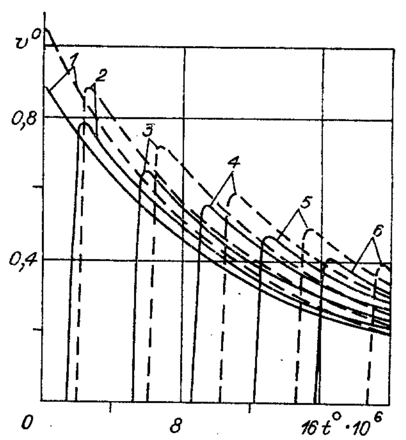

Fig. 3

the result in [2], no shock wave is observed here. The speed of propagation of the compression wave coincides with the speed of sound in the model of the medium without viscosity, the equation of compressibility of which is given by the expression (15):

$$\dot{R} = \left[\rho_0\left(\frac{\alpha_1}{E_D} + \sum_{i=2}^{3}\frac{\alpha_i}{\rho_{i0}c_{i0}^2}\right)\right]^{-1/2}. \tag{22}$$

In Figs. 1-3 the dimensionless quantities p^0, ε^0, v^0 are shown as functions of the dimensionless variable $\bar{t}$ at various sections of the medium. Curves 1-6 correspond to distances $\bar{r} = (0;\ 2;\ 6;\ 10;\ 14;\ 18)\cdot 10^{-6}$, the solid curves give the pressure, deformation, and speed in the viscous medium with $\alpha_1 = 0.69$, $\alpha_2 = 0.01$, $\alpha_3 = 0.3$, while the dashed curves are for $\alpha_1 = 0.6$, $\alpha_2 = 0.1$, $\alpha_3 = 0.3$.

It is evident from Figs. 1-3 and Eq. (22) that the pressure, deformation, and speed of the particles of the medium, as well as the speed of propagation of the wave, depend substantially on the content of various components in the medium. An increase in α_1 and, associated with it, a decrease in α_2 in the medium subject to cooling leads to an increase in the speed of the wave and to a decrease in deformation in the medium, a point noted in [3], where this dependence was confirmed experimentally for a number of frozen soils. In this connection, the pressure in the medium increases insignificantly.

Our basic aim in this paper is the construction of a model of a multicomponent medium with a variable coefficient of viscosity and the solution of the problem of propagation into this medium of a planar wave produced by nonstationary shock loading. The author's determination of η as the coefficient of viscosity of ice, unlike the situation in [2] where the coefficient of viscosity characterizes the properties of a multicomponent medium in the large, follows from a comparison of the fundamental equation of compressibility of a viscoelastic medium, as expressed in [2], and the equation given in [6] resulting from a series of experiments, and characterizes the viscous properties of only one component (in this case, ice). In the general case, the coefficient of viscosity of a medium depends on various factors: the porosity of the medium [2], the physicomechanical properties of its components, the form of and the time of the applied loading. Along with these factors, the compressibility and viscosity of frozen soils depend on the characteristics of the soils in the unfrozen state [3], as well as on the ice—water phase transition. One can state that the contribution from this last factor will differ in frozen soils of diverse structure. However, the effect of this phase transition (for lack of the necessary experimental data) is not considered in the present paper; it is assumed that the coefficient η introduced here defines the viscous properties of the medium in the large. In this connection, Eq. (3) describing the behavior of ice under the action of a load enables us to apply the method presented in [2] to construct a model of a multicomponent medium and to obtain a set of equations describing the process of wave propagation in this medium.

We note, in conclusion, that the main effect caused by a degeneracy in the viscosity coefficient at the front of the wave is the instantaneous smearing-out of the shock wave in the medium (converting it into a continuous wave of compression).

LITERATURE CITED

1. G. M. Lyakhov, "Shock waves in multicomponent media," Izv. Akad. Nauk SSSR, Mekh. Mashinostr., No. 1 (1959).
2. G. M. Lyakhov, Waves in Soils and Porous Multicomponent Media [in Russian], Nauka, Moscow (1982).
3. G. M. Lyakhov and G. B. Frash, "Blast waves in frozen soils," Zh. Prikl. Mekh. Tekh. Fiz., No. 6 (1983).
4. V. A. Fomin, A Study of Ice Fracture under High Deformation Rates [in Russian], Aerodynamic and Geocosmic Studies, MFTI, Moscow (1984).
5. V. A. Fomin and V. N. Rodionov, "On the dissipation of mechanical energy on ice," Dokl. Akad. Nauk SSSR, 285, No. 6 (1985).
6. L. Hald and N. Sinha, "Rheological behavior of ice under small deformations," in: Physics and Mechanics of Ice [Russian translation], Mir, Moscow (1983).
7. N. K. Sinha, "Rheology of columnar-grained ice," Exp. Mech., 18, No. 12 (1978).
8. N. E. Haskin, "Method of characteristics for the solution of equations of one-dimensional unsteady flow," in: Computational Methods in Hydrodynamics [Russian translation], Mir, Moscow (1971).

STRAIN-HARDENING OF STEEL BY DYNAMIC UNIAXIAL TENSION

A. G. Ivanov, A. I. Korshunov,
A. M. Podurets, V. A. Ryzhanskii,
and N. A. Yukina

UDC 539.375.5

As is well known, many structural materials are sensitive to loading history to some extent. One manifestation of such sensitivity is the strain-hardening of steel as a result of plastic deformation — so-called work-hardening. It was shown in [1] that work-hardening increases with an increase in strain rate. It was established in [2] that in the uniaxial tension of austenitic steel 12Kh18N10T in different rate regimes, the mechanical properties of the steel are significantly affected by factors related to its loading history (particularly relaxation processes and dynamic work-hardening).

The present study (a continuation of [2]) is devoted to examination of the changes in the physicomechanical properties of steel 12Kh18N10T as a result of its dynamic tension.

The material for our study was taken from a cylindrical shell (outside radius R_0, thickness $0.0246R_0$, length $4R_0$) welded from steel plates which were quenched and cooled in air. The shell was filled with water and was twice loaded by a spherical charge of high explosive (HE) detonated at the center. In each explosion, the charge itself was placed at the geometric center of the shell. The tests were conducted in open air. The temperature of the water-filled shell in the tests was (293 ± 5) K. The shell was deformed into a box shape as a result of the explosions. Measurement showed that the strains of the shell were close to uniaxial: with radial expansion, its considerable circumferential tension (about 40% in the central cross section) was accompanied by thinning and slight (maximum of about 3% in the same section) contraction along the generatrix. The results of high-speed photographs taken in the tests by the shadow method [3] showed that the shell pulsated slightly as it expanded (due to the action of compression waves circulated in the water, which is typical of underwater explosions [4]).

The test specimens were cut in three shell regions located at different sites and, thus, characterized by different loading histories: in the region of the central cross section A, closest to the center of the explosion (where the strain rate was therefore the highest), the circumferential plastic strain ε_0 was about 37%; in region B, located between region A and the edge of the shell, $\varepsilon_0 \simeq 17\%$; in region C at the edge of the shell, where the strain rate was lowest, $\varepsilon_0 \simeq 2\%$. The machining performed during cutting of the semifinished products and preparation of the specimens was done in regimes which kept it from affecting the properties of the material.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 133-140, November-December, 1987. Original article submitted June 4, 1986.